

**An experimental assessment of the
“Gibbs-Energy and Empirical-Variance” estimating equations
(via Kalman smoothing) for Matérn processes**

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The problem of estimating (from n noisy observations of a single realization, at known sites) the parameters of a centered stationary Gaussian process whose autocorrelation function belongs to the Matérn class appears in many contexts (e.g. [1, 2, 3]). The recently proposed CGEM-EV method [4] only requires the computation of several conditional means, at the observation sites, corresponding to candidate values for the Matérn parameters. In dimension 1 and when the “Matérn differentiability” parameter is fixed to $k + 1/2$ with k integer (an often-used value is $k = 0$ or $k = 1$, see e.g. [3], [1], [6], [7]), each of these conditional means reduces to a Kalman smoothing.

An R implementation of CGEM-EV for this context is presented : it is built on the R-package `dln` [5]. It proves to be quite fast, even for high-frequency sampling (e.g. $n = 8196$), and an empirical comparison with the classical maximum likelihood estimator confirms the near-efficiency results of [4].

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