

Maximum likelihood conjoint measurement: From GLM to GAM

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Conjoint measurement [5] is a psychophysical paradigm in which an observer is presented with pairs of stimuli varying independently along several dimensions and is required to order them according to one of those dimensions. Considering a task with two dimensions, A and B , for which the judgment relates to A , the internal response to each stimulus can be modeled as:

$$\psi^A = \psi_A^A(\phi^A) + \psi_B^A(\phi^B) + \psi_I^A(\phi^A, \phi^B) + \epsilon, \quad (1)$$

where ψ^A is the perceptual response to the stimulus for dimension A , ψ_X^A is the contribution of dimension X to this response, ϕ^X is the physical level of the stimulus for dimension X , and $\epsilon \sim N(0, \sigma^2)$ is judgment noise assumed to follow a standard Gaussian distribution. The judgment noise takes into account that observers may not make the same response to the same stimulus pair when the perceptual differences are small.

Equation (1) is called the “saturated model” because it incorporates to the judgments the contributions of both dimensions A and B and their possible interaction. The need for the interaction term can be evaluated with a likelihood ratio test, as the simpler additive model without that term is nested within it. An independent model can be similarly tested in which only one of the dimensions makes a significant contribution to the observer’s judgments.

The decision rule between stimuli 1 and 2 is given (for the saturated model) by:

$$\begin{aligned} \Delta_{12} &= \psi_1^A - \psi_2^A \\ &= [\psi_A^A(\phi_1^A) - \psi_A^A(\phi_2^A)] + [\psi_B^A(\phi_1^B) - \psi_B^A(\phi_2^B)] + [\psi_I^A(\phi_1^A, \phi_1^B) - \psi_I^A(\phi_2^A, \phi_2^B)] + \epsilon, \end{aligned} \quad (2)$$

where Δ_{12} is the noise-contaminated decision variable and ψ_1^A and ψ_2^A are the perceptual responses to stimuli 1 and 2, respectively, when judging dimension A . The inclusion of noise in this model makes it possible to estimate the respective contributions of each dimension (the latent responses, ψ_Y^X) by maximum likelihood, producing Maximum Likelihood Conjoint Measurement (MLCM) [2]. Knoblauch and Maloney [3] have also shown that this analysis can be reformulated as a special case of the Generalized Linear Model (GLM) [6] with a Bernoulli distribution. These analyses are simplified in R using the **MLCM** package [4].

In most applications, the number of levels tested along each dimension is small, and they are treated as categorical variables, which ignores the continuous nature of the physical scales

and underlying psychophysical functions. If a sufficient number of levels is tested for each dimension, this issue can be addressed by reformulating the problem as a Generalized Additive Model (GAM). A GAM is a penalized GLM resulting in a smooth curve defined by a regression spline where the complexity is constrained by a criterion related to cross-validation [6]. We implement the smooth MLCM model using the `mgcv` package [6]. Each term in brackets on the right side of Equation 2 is represented as a linear functional using the `by` argument of the smooth terms to specify a 2-column matrix of weights, $(1, -1)$, for each of the two terms within the brackets.

We demonstrate the method using data from a gender comparison task in which the voices and faces of video stimuli varied through morphing along a gender continuum over nearly 20 levels. On each trial a pair of stimuli was presented with the gender scale values of the voice and face independently and randomly assigned to each image. In different sessions the observer was required to choose which face, voice or stimulus was more masculine (or feminine). Observers were tested over several sessions on 1500 stimulus pairs randomly sampled from the full set of pairs over both dimensions. We compare the results from the GAM and GLM analyses for the full stimulus set, as well as a GLM analysis of a stimulus set restricted to only 5 levels along each dimension and exhaustively sampled that we have previously shown to give similar results when an equal number of trials is tested [1].

The use of GAM models is a promising approach for characterizing and testing the contributions of different stimulus dimensions to perception.

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